The math behind the
mobius preview control panel java applet

Richard A. DeVenezia
March 2000
www.devenezia.com

Abstract

This discussion covers topics involved in the implementation of my java applet that displays a mobius strip and allows a user to manipulate various parameters regarding the construction and display of the strip. This document covers some topics of my mobius strip model, but may contain information about features not implemented, and the applet definitely contains features not discussed.

Mobius Geometry

In the purist sense, a mobius strip is an object in space with only a width and circumferential measure. The strip becomes more interesting by adding the element of thickness to it. A thickened mobius strip can be constructed by laying out a strip, twisting it and curling the strip until the ends meet.

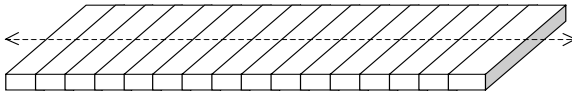## Strip Construction

```
Strip (L, W, H, Nₛ)
L - Length
W - Width of cross section
H - Height of cross section
Nₛ - Number of equisized segments along length
    (N-1 equisized divisions)
    (N+1 cross sections at equispaced intervals)
```
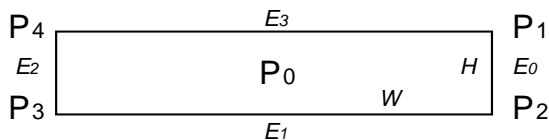

A strip is composed of segments with a rectangular cross section. The cross sections, called slices, are evenly spaced.

Cross sectional slice with reference points and edges

The slice is the basic unit of construction, segments are constructed by displaying faces created by connecting like named slice edges (the slice itself can be a face.)

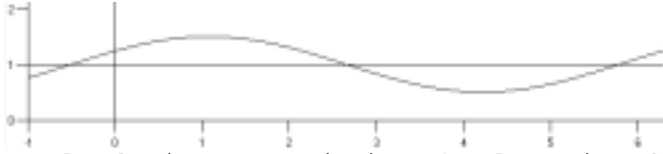## Sinusoidal Variation of a Parameter Along Length

```
Wave (P, M, N_c, K)
P - Parameter base line value
M - Percentage variation of base line value (relative amplitude of wave)
N_c - Number of half-cycles of variation to occur along the length
K - Sine wave offset at start of length
```

Sample showing 50% variation of value 1 in 2 half cycles.
$P=1, M=50, N_c=2, K=0.5$

$P' = P (1 + M/100 \cdot \sin (\theta \cdot N_c \cdot \pi + K))$
where $\theta = 2\pi \cdot n / N_s$, $n=0...N_s$

Sinusoidal variation in a static form adds an element of waviness.
Timewise variation of a parameter adds an element of organicness.

Cross Section Along Length Construction

Slice (W_0, W_1, W_2, W_3)
W_n - Description of how the size of edge E_n varies sinusoidally along the length

If a W_i is not present, the edge i will be either W wide or H high.
When a W_i is present, edge size is distributed symmetrically about the edge center.
Assume width is x and height is y:
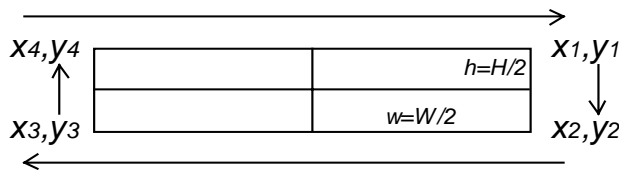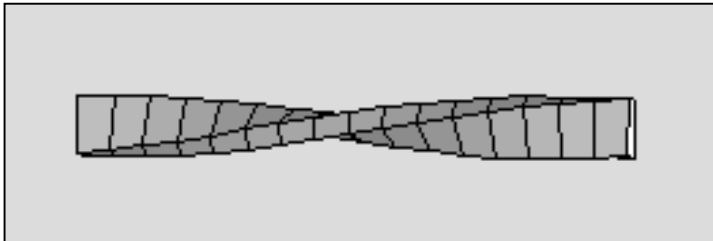

for $n=0...N_s$

$y_1 = \text{Wave} (h, W_0)$ $y_2 = -y_1$
$x_2 = \text{Wave} (w, W_1)$ $x_3 = -x_2$
$y_3 = \text{Wave} (-h, W_2)$ $y_4 = -y_3$
$x_4 = \text{Wave} (-w, W_3)$ $x_1 = -x_4$

Several other parameters could be allowed to vary along the length. Slice spacing,
edge tops and bottoms come to mind. The entire variation process could be handed off
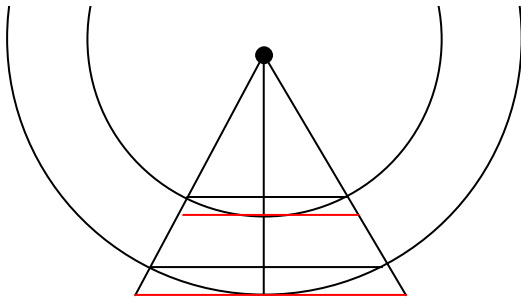to a model.

Strip Twist Construction

Twist (N)

N - Number of half-twists that should be applied to the strip before curling

Sample showing N=1

Strip Curling

Scribing


```
Inscribing - Vertices are laid out such that they lie on the circle
Circumscribing - Vertices are laid out such that the segment center lies on the circle
```

Inscribing

$$x_n = r\cos(\theta_n); \qquad \theta_n = 2\pi \cdot n/N_s, \ n=0...N_s$$
$$y_n = r\sin(\theta_n); \qquad \theta_n = 2\pi \cdot n/N_s, \ n=0...N_s$$

Circumscribing

$$x_n = r'\cos(\theta_n); \qquad \theta_n = 2\pi \cdot n/N_s, \ n=0...N_s$$
$$y_n = r'\sin(\theta_n); \qquad \theta_n = 2\pi \cdot n/N_s, \ n=0...N_s$$
$$r' = (r + \delta), \text{ the radius of circumscription}$$
$$\theta = 2\pi/N_s$$
$$\cos(\theta/2) = r / r'$$

hence, $r' = r / \cos(\pi/N_s)$

As N_s increases this differentiation becomes insignificant

## *Curl Construction*

```
Curl (F, Cn, Cn0)
```

F - Curl inner radius factor, 0..1. At 0 the strip length is distributed to the
    outside of the curl, at 1 the length is distributed to the inside of the curl.

C_n - Number of segments to curl

C_{n0} - Segment number where curling starts.

S - Curling style, 0 for start forward, 1 for start alternate.

To allow interesting effects, number of segments curled is a parameter. When less
than the total number of segments the resulting object is not visually closed.
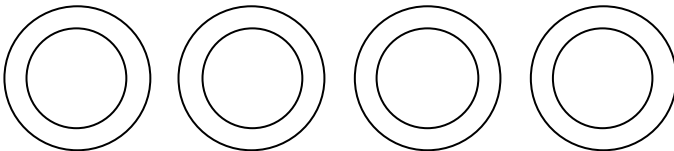
Visual Style	C_n	C_{n0}	S
Wrap-Around	varies	fixed	0
Roll-Up	varies	varies with C_n	0
Hug	varies	fixed	1

Inner radius r of a strip with no twists is $r = L / 2\pi - F \cdot h$

Presently wrap-around and roll-up are implemented.

Timewise Variation of Parameters

This element of variation produces the most interesting effects. Timewise variation
can create an impression of organic motion. Control of timed variation is done using
the RepeatingLoop control.

```
Start #      ● Repeat    0    10   30
☑ Cycle  ○ Bounce      ▽
```
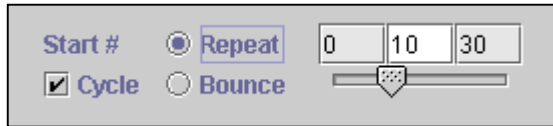
Start # – This varies depending on the parameter. It is simply a label telling you
          what parameter is being affected.
Cycle - Checked to run the loop, Unchecked to pause the loop
Repeat/Bounce - How the loop repeats when the current value meets a boundary value
0,10,30 - minimum value, current value, maximum value. Varies per parameter. You can
          place any value in these fields to change the range of the looping
Slider – Visual cue of the current value. Grab it and slide it to change the current
          value. Focus it and use the left and right arrows to go down or up one at a
          time. Drag the slider in the opposite direction of the running direction to
          reverse the direction. How fast/far you drag the slider changes how fast
          the slider will run on its own.